

Lecture 2

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Revision

$$\vec{\nabla} = \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k}$$

nabla operator

del operator

delta operator

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\vec{\nabla} \phi \quad \text{grad } \phi \quad (\text{vector})$$

$$\vec{\nabla} \cdot \vec{F} \quad \text{divergence} \quad \text{div } \vec{F} \quad (\text{scalar})$$

$$\vec{\nabla} \times \vec{F} \quad \text{curl} \quad \text{curl } \vec{F} \quad (\text{vector})$$

$$\vec{\nabla} \cdot \vec{V} = 0 \Rightarrow \text{solenoidal field} \quad (\text{أنيوبي})$$

$$\vec{\nabla} \times \vec{F} = 0 \Rightarrow \text{irrotational} \quad (\text{غير دوڑانی})$$

$$\vec{\nabla} (\phi_1 + \phi_2) = \vec{\nabla} \phi_1 + \vec{\nabla} \phi_2$$

الخطاب، لا التبعيات

المبدأ

$$\vec{\nabla} (\phi_1 + \phi_2) = \vec{\nabla} \phi_1 + \vec{\nabla} \phi_2$$

$$\left(\frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k} \right) (\phi_1 + \phi_2)$$

$$\vec{\nabla} \cdot (\vec{A}_1 + \vec{A}_2) = \vec{\nabla} \cdot \vec{A}_1 + \vec{\nabla} \cdot \vec{A}_2$$

$$\vec{\nabla}_x (\vec{A}_1 + \vec{A}_2) = \vec{\nabla}_x \vec{A}_1 + \vec{\nabla}_x \vec{A}_2$$

Ex.

$$\vec{\nabla} \cdot (\phi \vec{A}) = \phi \vec{\nabla} \cdot \vec{A} + \vec{A} \cdot \vec{\nabla} \phi$$

المشتق

Ex.

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

$$\therefore \vec{A} = A_1 \vec{i} + A_2 \vec{j} + A_3 \vec{k}$$

$$\text{Ex. } \vec{\nabla}_x \vec{\nabla} \phi = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \end{vmatrix} = \vec{0}$$

Ex.

$$\vec{\nabla} \cdot \vec{\nabla} \phi = \left(\frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k} \right) \cdot \left(\frac{\partial \phi}{\partial x} \vec{i} + \frac{\partial \phi}{\partial y} \vec{j} + \frac{\partial \phi}{\partial z} \vec{k} \right)$$

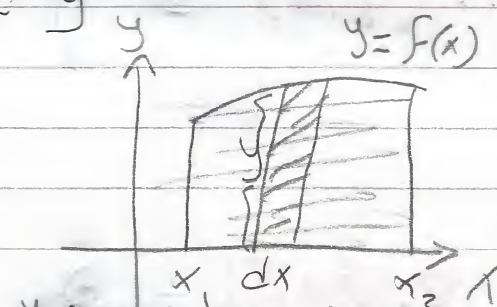
$$= \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \right) = \Delta \phi$$

Line integral التكامل الخطي

independant variable x
dependant variable y

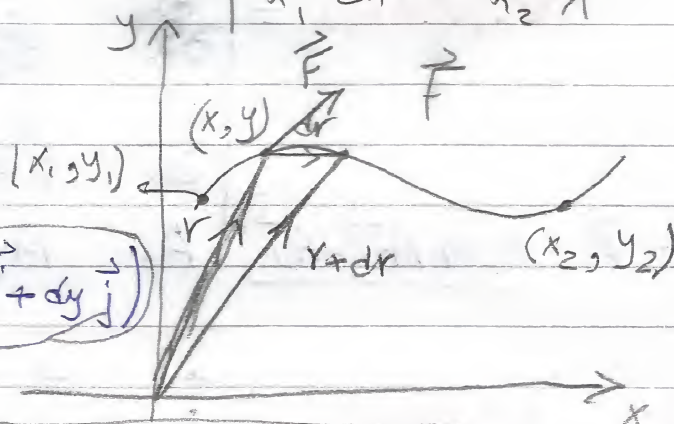
Riemann integral

$$\int y(x) dx$$



$$\int_{(x_1, y_1)}^{(x_2, y_2)} \vec{F} \cdot d\vec{r} \equiv$$

$$= \int_{(x_1, y_1)}^{(x_2, y_2)} (F_1 \vec{i} + F_2 \vec{j}) \cdot (dx \vec{i} + dy \vec{j})$$



$$\boxed{\vec{r} = x\vec{i} + y\vec{j} \quad d\vec{r} = dx\vec{i} + dy\vec{j}}$$

$$F_1 = F_1(x, y) \quad F_2 = F_2(x, y)$$

$$\int_{(x_1, y_1)}^{(x_2, y_2)} F_1 dx + F_2 dy$$

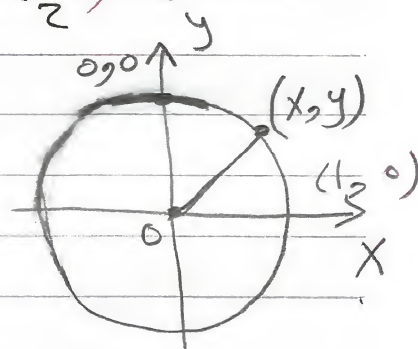
Evaluate The Line integral $\int 2xy dx + (x^2 + y^2) dy$ on the curve given by The equation

$$(x = \cos t) \& (y = \sin t); (0 \leq t \leq \frac{\pi}{2})$$

Sol.

$$\int 2xy dx + (x^2 + y^2) dy$$

$$= \int 2xy dx + \int x^2 + y^2 dy$$



التحويل ~~من~~ ليكون المتغير x

$$= 2 \int_1^0 x \sqrt{1-x^2} dx + \int_0^1 dy$$

$$x^2 + y^2 = 1$$

$$y = \pm \sqrt{1-x^2}$$

الكل في المتغير

$$x = \cos t$$

$$dx = -\sin t dt$$

$$y = \sin t$$

$$dy = \cos t dt$$

$$I = \int_0^{\frac{\pi}{2}} (2 \cos t \sin t (-\sin t dt) + \cos t dt)$$

$$= -2 \int_0^{\frac{\pi}{2}} \cos t \sin^2 t dt + \int_0^{\frac{\pi}{2}} \cos t dt$$

$$= -2 \int_0^{\frac{\pi}{2}} \sin^2 t \, d \sin t + \int_0^{\frac{\pi}{2}} \cos t \, dt$$

$$= \frac{-2}{3} (\sin^3 t) \Big|_0^{\frac{\pi}{2}} + (\sin t) \Big|_0^{\frac{\pi}{2}}$$

$$= \frac{-2}{3} + 1 = \frac{1}{3}$$

تبدیل

$$\frac{d}{dt} \sin t = \cos t$$

$$d \sin t = \cos t \, dt$$